Design of Compact Composite Right-Left-Handed Transmission Line Unit Cells

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Abstract— A new unit cell composite right/left-handed (CRLH) metamaterial using microstrip technology with coupled line is proposed. It is simple and has a small size. The proposed unit cell can be designed in several frequency bands based on its geometrical parameters. The transition from LH to RH shows the balanced condition without the presence of stop band. Two designs for two different frequency bands have been carried out. The frequency band of the first design is about 320 MHz and the band of the second design is about 1.3 GHz. This unit cell is fabricated and its measurement results are shown. Good agreement has been achieved between the theoretical and the experimental results.

Keywords- Metamaterials; Composite right/left-handed; Microstrip; Bloch Impedance; CRLH; FIT.

I. INTRODUCTION

In recent years, an increased interest appeared in the scientific community for the study of metamaterials. Metamaterials are artificially fabricated materials having electromagnetic properties not present among natural ones. The most important properties of the metamaterials are the Negative permittivity (ε) and permeability (μ), due to this marvelous properties, it is used for implementation of some applications, such as the enhancement performance of patch antennas [1], and directional couplers [2].

Metamaterials are periodic structures formed by symmetrical cells. Some designs of these unit cells were suggested using conventional components, such as microwave short-open stubs, coupled lines [3], interdigital capacitors, split ring resonators [4], and spiral inductors [5].

The most common microstrip type CRLH transmission line is designed using interdigital capacitors to achieve the required capacitance, and shorted stub inductors to achieve the required inductance [2]. The microstrip coupled line unit cells [6] and the even/odd mode CRLH unit cells [7] have relatively large sizes.

In this paper; the presented composite right-left-handed unit cells have short stubs, coupled line and ground defects, it has a small size and easiness for design.

The paper is organized as follows: section (II) includes the theory and microstrip implementation. Section (III), includes both the simulation and the measurement results, section (IV) includes conclusions and the more relevant references.

II. THEORY

A. CRLH TL

The propagation constant of the CRLH TL, can be obtained as follows [8]

\[
Z' = j(\omega L_R - \frac{1}{\omega C_L})
\]  

(1.a)

\[
Y' = j(\omega C_R - \frac{1}{\omega L_L})
\]  

(1.b)

Where \(Z'\) and \(Y'\) are the per-unit length impedance and admittance, respectively, and the shunt and series resonance frequencies are given by:

\[
\omega_{\text{shunt}} = \frac{1}{\sqrt{L_R C_R}}
\]  

(2.a)

\[
\omega_{\text{series}} = \frac{1}{\sqrt{L_L C_L}}
\]  

(2.b)

Using equations 1 and 2 the complex propagation constant \(\beta(\omega)\) is given by:

\[
\beta(\omega) = s(\omega) \sqrt{\omega^2 L_R C_R + \frac{1}{\omega^2 L_L C_L} - \left(\frac{L_R}{L_L} - \frac{C_L}{C_R}\right)}
\]  

(3)

Where

\[
s(\omega) = \begin{cases} 
-1 & \text{if } \omega < \omega_{\text{series}} = \min(\omega_{\text{series}}, \omega_{\text{shunt}}) \\
+1 & \text{if } \omega < \omega_{\text{shunt}} = \max(\omega_{\text{series}}, \omega_{\text{shunt}}) 
\end{cases}
\]  

(4)

In the balanced case, the series and the shunt resonances are exactly equal to each other at a certain frequency [9], and the transition from LH and RH occurs at the transition frequency \(\omega_0\) which is given by:

\[
\omega_0 = \sqrt{\omega_{\text{series}}\omega_{\text{shunt}}} = \frac{1}{\sqrt{\frac{L_R}{L_L} + \frac{C_L}{C_R}}}
\]  

(5)

B. Proposed Unit Cell

The proposed unit cell is shown in Fig. 1. It consists of two short stubs \((l_2 < \lambda/4)\), open stubs coupled line \((l_3 < \lambda/4)\), and slot line, where \(p\) is the length of the unit cell. The equivalent circuit of the proposed unit cell is shown in Fig. 2,
in this equivalent circuit, the short stubs provide the LH shunt inductance \( L_s \), the gaps between the coupled transmission line provide the LH series capacitance \( C_L \). The coupled line represents two open stubs \( (l_s < \lambda/4) \). It has a parasitic shunt capacitance \( (C_{slot}) \), that increases the shunt capacitance of the transmission line. To overcome the effect of this shunt capacitance, defected ground slot lines have been introduced in the ground plane as shown in Fig. 1 (b). The balanced condition has been obtained and controlled by these defected slot lines as shown in Fig. 5. The defected ground slot line is represented in the equivalent circuit by a tuned circuit \[10\] which is consisting of the series inductance \( L_{slot} \), and the shunt capacitance \( C_{slot} \), as shown in Fig. 2. The equivalent circuit of CRLH unit cell, it consists of two \( \pi \) network symmetry, convert the \( \pi \) network to \( T \) network by using the ABCD parameters two-Port circuits \[11\], that is shown in Fig. 3. \( Z \) and \( Y \) are the series impedance and shunt admittance.

Figure 1. Proposed microstrip CRLH unit cell: (a) Top layer unit cell (b) Bottom layer ground defects.

Figure 2. Equivalent circuit of the proposed CRLH unit cell.

Figure 3. Equivalent circuit of the series impedance and the shunt admittance.

C. Analysis

The periodic boundary conditions (PBCs) are applied to the unit cell represented by its \([ABCD]\) matrix. The ABCD parameters of the unit cell is given as follows \[8\]:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & Z/4 \\
0 & 1/2
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
1 & Z/2
\end{bmatrix} \begin{bmatrix}
0 & 1 \\
1/2 & Z/4
\end{bmatrix} = \begin{bmatrix}
1 + ZY/2 \\
0 & Y
\end{bmatrix} \begin{bmatrix}
1/Z & Z(1 + ZY/2) \\
1 & 1 + ZY/2
\end{bmatrix}
\]

That the phase constant \( \beta \) for the symmetric CRLH unit cell is

\[
\beta = \frac{1}{p} \cos^{-1} \left( 1 + \frac{ZY}{2} \right) = \frac{1}{p} \cos^{-1}(A)
\]

The scattering matrix is the voltage wave incident on the ports to voltage wave reflected form the ports. By using the relationship between scattering matrix and \([ABCD]\) matrix [9].

\[
A = \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}
\]

\[
B = \frac{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}{2S_{21}}
\]
The phase constant is given by

\[ \beta = \cos^{-1}\left(\frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}\right) \]  

(9)

D. Microstrip Implementation

The presented equivalent circuit model for the unit cell of CRLH (Fig. 2) can be implemented using the microstrip technology. It should be noted that the values of shunt inductance \( L_s \), capacitance and inductance of host transmission line, are calculated according to the equations given in [12].

\[ L(nH) = 2 \times 10^{-6} - \ln\left(\frac{1}{w + t}\right) + 1.193 + \frac{0.2235 \sqrt{w t}}{t} \times k \]  

(10)

\[ kg = 0.57 - 0.145 \ln\left(\frac{w}{t}\right) \]  

(11)

\[ C(\mu f) = 16.67 \times 10^{-6} \sqrt{\frac{w t}{\varepsilon_r}} \]  

(12)

Where \( w \) and \( l \) are the width and length of the microstrip line, respectively, \( t \) is the thickness of the strip, \( h \) is the thickness of substrate, \( Z_0 \) is the characteristic impedance and \( \varepsilon_{re} \) is the effective dielectric constant.

The series capacitance \( C_L \) calculated using the coupling capacitance for the coupled line.

The realization for slot lines can be a set of short-circuited connected in parallel, and can be calculated as short stub.

III. SIMULATION AND MEASUREMENT RESULTS

A. Dispersion diagram:

The simulation was carried out using CST microwave studio that implements finite integration technique FIT in time domain [13]. The three unit cells cascaded are simulated on a Rogers RO3006 substrate with a dielectric constant of 6.15 and the thickness of 0.635 mm. Table I shows the parameters of the CRLH unit cells, these parameters are obtained for a system impedance of 50\( \Omega \). The 0.5 mm and 0.2 mm are the dimensions of the radius via hole, and the space between the coupled lines, respectively. These dimensions are available for implementation within our fabrication facility.

![Figure 4](image_url)

**Figure 4.** Dispersion relation for the balanced CRLH unit cell.

**Design Wide band Unit cell**

The unit cell is simulated on a Rogers RO3006 substrate with a dielectric constant of 6.15 and the thickness of 0.635 mm. Table II shows the parameters of the simulation unit cell CRLH for wide band region about 1.3 GHz.

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>0.5</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>2.7</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>4.1</td>
</tr>
<tr>
<td>( l_3 )</td>
<td>6.25</td>
</tr>
<tr>
<td>( S )</td>
<td>0.2</td>
</tr>
<tr>
<td>( l_{slot} )</td>
<td>3.498</td>
</tr>
<tr>
<td>( w_{slot} )</td>
<td>0.25</td>
</tr>
<tr>
<td>Radius of via holes</td>
<td>0.5</td>
</tr>
<tr>
<td>( P )</td>
<td>5.6</td>
</tr>
</tbody>
</table>

### Table I. Design Specification of Unit Cell

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>1</td>
</tr>
<tr>
<td>( W_2 )</td>
<td>1</td>
</tr>
</tbody>
</table>
TABLE II. DESIGN SPECIFICATION OF UNIT CELL

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>Value</th>
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<tbody>
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<td>$W_1$</td>
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</tr>
<tr>
<td>$W_2$</td>
<td>1</td>
</tr>
<tr>
<td>$W_3$</td>
<td>0.5</td>
</tr>
<tr>
<td>$l_1$</td>
<td>2.7</td>
</tr>
<tr>
<td>$l_2$</td>
<td>3</td>
</tr>
<tr>
<td>$l_3$</td>
<td>3.5</td>
</tr>
<tr>
<td>$S$</td>
<td>0.2</td>
</tr>
<tr>
<td>$l_{slot}$</td>
<td>4.925</td>
</tr>
<tr>
<td>$w_{slot}$</td>
<td>0.25</td>
</tr>
<tr>
<td>Radius of via holes</td>
<td>0.5</td>
</tr>
<tr>
<td>$P$</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Figure 5. Shows the dispersion characteristics, and the band regions from 3.7 to 5.08 GHz. The LH region is to be between 3.7GHz and the transition frequency 4.24GHz, and the RH region is to be between 4.24GHz and 5.08GHz. The transition from LH to RH shows the balanced case without the presence of stop band.

C. Effect of the ground defects:

The length of the slot line effects on the structure, for controlling the balance case. Fig 7. Shows the unit cell CRLH unbalance case with three different lengths of the slot lines. The stop band presence with unbalance condition, the stop band increases with increases the length of the slot line.

D. Experimental Results

For experimental verification, the simulated CRLH three unit cells are fabricated. Figure 8, shows a photograph of the fabricated unit cells. Figure 9, Shows the comparison between the simulated and measured scattering parameters of the three unit cells.
The compact size of the design and existence of the two layers have represented a challenge for fabrication. Intercontinental microwave is used to measure the results. A small shift between the measured and simulated results is present due to low fabrication techniques.

IV. CONCLUSION

The CRLH unit cell of the microstrip technology was proposed. It has a simple design and compact size. The proposed unit cell can be designed in several bands based on its geometrical parameters, with the balanced case between LH and RH bands. The unit cell is fabricated and its measurement results are shown. Good agreement has been achieved between theoretical and experimental results.

REFERENCES